Carboxysome partitioning error

Hernan G. Garcia

September 21, 2015

In class we calculated the error in partitioning of N_{mother} carboxysomes in the mother cell upon division into two daughter cells each with a number of carboxysomes N_1 and N_2. We used a simple thought experiment of a single particle diffusing in 1D to justify that the variance in N_1 is given by

$$\mathrm{var}(N_1) = \langle (N_1 - \langle N_1 \rangle)^2 \rangle = pqN_{mother}, \tag{1}$$

where p and q are the probabilities of a carboxysome going to daughter cell 1 or 2, respectively. We can open up the middle term to get

$$\mathrm{var}(N_1) = \langle N_1^2 - 2N_1\langle N_1 \rangle + \langle N_1 \rangle^2 \rangle. \tag{2}$$

We further simplify this expression making use of the fact that the average of a sum is just the sum of the individual averages and obtain

$$\langle N_1^2 - 2N_1\langle N_1 \rangle + \langle N_1 \rangle^2 \rangle = \langle N_1^2 \rangle - \langle 2N_1 \langle N_1 \rangle \rangle + \langle N_1 \rangle^2. \tag{3}$$

Next, we examine the middle term on the right-hand-side of the previous equation. Note that $\langle N_1 \rangle$ is a number that can be factored out of the average it's in. As a result we get

$$\langle N_1^2 \rangle - \langle 2N_1 \langle N_1 \rangle \rangle + \langle N_1 \rangle^2 = \langle N_1^2 \rangle - 2\langle N_1 \rangle \langle N_1 \rangle + \langle N_1 \rangle^2 \tag{4}$$

which can be further simplified into

$$\mathrm{var}(N_1) = \langle N_1^2 \rangle - \langle N_1 \rangle^2 = pqN_{mother}. \tag{5}$$

Alternatively, this expression can be written as

$$\langle N_1^2 \rangle = pqN_{mother} + \langle N_1 \rangle^2. \tag{6}$$

In the carboxysome experiment discussed in class we were interested in calculating the average error in partitioning given by

$$\langle (N_1 - N_2)^2 \rangle = \langle (N_1 - (N_{mother} - N_1))^2 \rangle, \tag{7}$$

where we have made use of the fact that $N_1 + N_2 = N_{mother}$. We now open the parenthesis inside the average and calculate its square

$$\langle (N_1 - N_2)^2 \rangle = \langle (2N_1 - N_{mother})^2 \rangle = \langle 4N_1^2 - 4N_1 N_{mother} + N_{mother}^2 \rangle. \quad (8)$$

Once again we make use of the fact that the average of a sum is the sum of the averages to further simplify this expression and obtain

$$\langle 4N_1^2 - 4N_1 N_{mother} + N_{mother}^2 \rangle = 4\langle N_1^2 \rangle - 4N_{mother}\langle N_1 \rangle + N_{mother}^2. \quad (9)$$

Since we know that $\langle N_1 \rangle = pN_{mother}$ we can further simplify this expression. From here on we will also assume that $p = q = 1/2$ which leads to

$$\langle (N_1 - N_2)^2 \rangle = 4\langle N_1^2 \rangle - 2N_{mother}^2 + N_{mother}^2 = 4\langle N_1^2 \rangle - N_{mother}^2. \quad (10)$$

Finally, we use eqn. 6 to replace $\langle N_1^2 \rangle$ and obtain

$$\langle (N_1 - N_2)^2 \rangle = 4\left(\frac{1}{4}N_{mother} + \langle N_1 \rangle^2\right) - N_{mother}^2 = N_{mother} + 4\langle N_1 \rangle^2 - N_{mother}^2 \quad (11)$$

which, by replacing for the known value of $\langle N_1 \rangle$, results in

$$\langle (N_1 - N_2)^2 \rangle = N_{mother}. \quad (12)$$

As a result, for the case of binomial partitioning, the square of the error in carboxysome partitioning between daughter cells should be given by the number of carboxysomes in the mother cell. Any deviation from this prediction would indicate that the partitioning is not a purely random process.