MCB261 solutions -- R. S. Zucker

1. The standard error of an estimate is equal to the standard deviation of that estimate divided by $\sqrt{N}$, where N is the number of observations. For a Poisson random variable x, the variance of x = mean of x. The variance (σ^2) is the square of the standard deviation, or S.D. = $\sqrt{\sigma^2}$. Therefore,

$$S.E._x = \frac{S.D._x}{\sqrt{N}} = \sqrt{\sigma^2/N} = \sqrt{m/N}$$

if m is the average of x.

To get $S.E._x/m = 0.1$, you need $S.E._x/m = 0.1 = \sqrt{1/mN}$, or you must make N observations, where $0.1^2 = 0.01 = 1/mN$, or $N = 100/m$. If m = 1, you must record 100 responses.

2. The equation for $\bar{p}$ that includes the effect of σ_p^2 is

$$\bar{p} = 1 - \frac{\sigma_V^2 - \sigma_N^2 - m\sigma_q^2}{mq^2} - \sigma_p^2/\bar{p} \quad (1),$$

where $\bar{p}$ includes correction for σ_p^2. Ignoring any variance in p gives

$$p = 1 - \frac{\sigma_V^2 - \sigma_N^2 - m\sigma_q^2}{mq^2} \quad (2).$$

In two pulse facilitation, $m_1 = 5$ and $m_2 = 10$. P_1 was estimated as 0.5 and p_2 as 0.75 ignoring σ_p^2 and using Eq. (2). Since $n = m/p$, we calculate that $n_1 = 10$ and $n_2 = 13.3$ (or 13). So both n and p appear to increase.

From Eq. (1) and (2), $\bar{p} = p - \sigma_p^2/\bar{p}$.

If $\sigma_p^2 = .25\bar{p}$, then $\bar{p}_1 = .5 - .25 = .25$ and $n_1 = 5/.25 = 20$
and $\bar{p}_2 = .75 - .25 = .5$ and $n_2 = 10/.5 = 20$,

and n did not really change at all!

In tabular form:

EPSP	#1	#2	
m	5	10	
p	0.5	0.75	ignoring σ_p^2
n	10	13	ignoring σ_p^2
$\bar{p}$	0.25	0.5	considering σ_p^2
n	20	20	considering σ_p^2

3. $C(r,t) = \frac{J}{2\pi D r} \operatorname{erfc} \frac{r}{2\sqrt{Dt}}$

A. $\operatorname{erfc}(0.2) = 0.78$ $\quad 0.2 = \frac{r}{2\sqrt{Dt}} \quad t = \frac{r^2}{4D(.2)^2}$

$r = 2\times10^{-6}$ cm $\quad D = 6\times10^{-6}$ cm²/s

$\boxed{T = 4\mu s}$ $\boxed{\text{steady state reached in 1 ms}}$

B. $J = \frac{i}{zF}$ $\quad z=2 \quad F = 10^5$ coul/mole $\quad i = \frac{1}{2}pA = 5\times10^{-13}A$

$c(r,\infty) = \frac{i}{2\pi zFDr} = \frac{5\times10^{-13}}{2\cdot\pi\cdot2\cdot10^5\cdot6\times10^{-6}\cdot2\times10^{-6}} = 3.3\times10^{-8}\,M/cm^3$

$= 3.3\times10^{-5}\,M/l$

$= \boxed{33\,\mu M}$

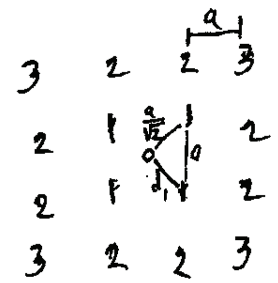
C. i) $c(r,\infty) = \frac{i}{2\pi zFB\cdot\frac{D}{BT}r} = \frac{iT}{2\pi zFDr}$ unaffected by B, multiplied by T so $100\,\mu M$

ii) $\operatorname{erfc}(x) = .78 \rightarrow x = .2 = \frac{r}{2\sqrt{Dt/BT}}$ $\quad (.2)^2 = \frac{r^2}{4Dt/BT}$

$t = \frac{r^2 BT}{4D(.2)^2} = \frac{(2\times10^{-6})^2(3)(100)}{(4)(6\times10^{-6})(.2)^2} = \boxed{1.25\,ms}$ $\boxed{\text{steady-state nearly reached in 1 ms}}$

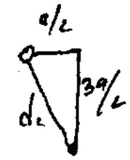
D. within $4\mu s$ as in A.

E.



$a = 50\,nm = 5\times10^{-6}\,cm$ Type 1: $d_1 = a/\sqrt{2}$

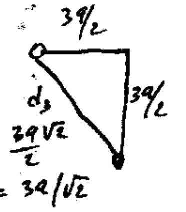
Type 2



$d_2 = \sqrt{\frac{a^2}{4} + \frac{9a^2}{4}} = \frac{\sqrt{10}\,a}{2}$

$= 1.58a$

Type 3



$= \frac{3a\sqrt{2}}{2}$

$= 3a/\sqrt{2}$

$d_3 = \frac{3}{\sqrt{2}}a$

$= 2.12a$

4 channels at $a/\sqrt{2} = .707a$ Type 1
8 channels at $1.58a$ Type 2
4 channels at $2.12a$ Type 3

Each channel contributes $Ca(r,t) = \frac{iT}{2\pi z F D r}\,\mathrm{erfc}\left(\frac{r}{2\sqrt{Dt/\beta T}}\right) = \frac{A}{r}\,\mathrm{erfc}(Br)$

$A = \frac{iT}{2\pi z F D} = \frac{5\times10^{-13}\cdot 3}{4\pi\cdot10^5\cdot 6\times10^{-6}} = 19.8\times10^{-14}\,\frac{M}{cm^3}\cdot cm = 1.98\times10^{-10}\,Molar\cdot cm$

$B = \frac{\sqrt{\beta T}}{2\sqrt{Dt}} = \frac{1}{2}\sqrt{\frac{3\cdot 100}{6\times10^{-6}\cdot10^{-3}}} = 1.1\times10^{5}\ /cm$

Contributions from all 16 channels summate

Type	r (cm)	Br	erfc(Br)	Ca(r,t) (μM)	n	n Ca(r,t) (μM)
1	3.54×10^{-6}	0.385	0.586	32.78	4	131.1
2	7.9×10^{-6}	0.869	0.219	5.49	8	43.8
3	1.06×10^{-5}	1.17	0.010	0.019	4	.6

$\boxed{175.5\,\mu M}$

More Than [Ca] 20 nm from 1 channel (~75 μM)

$= \frac{1.98\times10^{-10}}{2\times10^{-6}}\,\mathrm{erfc}(1.1\times10^{5}\cdot 2\times10^{-6})\ M$