	In	Out	$\text{Nernst}_i = 58 \log \frac{c^{out}}{c^{in}}$
K^+	140	5	−84 mV
Na^+	10	145	+67 mV
Cl^-	4	110	−83.5 mV

$P_{Na}/P_K = 0.019$

$P_{Cl}/P_K = 0.381$

1. The Goldman (or GHK) Equ:

$$V = 58 \log_{10} \frac{\sum_i (P_i c_i^{out} + P_i a_i^{in})}{\sum_i (P_i c_i^{in} + P_i a_i^{out})}$$

c_i = cations $z_i = +1$

a_i = anions $z_i = -1$

all permeabilities may be divided by P_K (i.e, divide numerator + denominator by P_K)

$$V = 58 \log_{10} \frac{K^{out} + \frac{P_{Na}}{P_K} Na^{out} + \frac{P_{Cl}}{P_K} Cl^{in}}{K^{in} + \frac{P_{Na}}{P_K} Na^{in} + \frac{P_{Cl}}{P_K} Cl^{out}}$$

$$= 58 \log_{10} \frac{5 + (0.019)(145) + (0.381)(4)}{140 + (0.019)(10) + (0.381)(110)} = -75.0 \text{mV}$$

2. See Table for Nernst_i values

3. No ions are at equilibrium. V_m is more negative than E_{Nernst} for Na, so Na^+ leaks in. V_m is more positive than E_{Nernst} for K and Cl, so K^+ leaks out and Cl^- (negative charge) leaks in. Since all ions are running down their concentration gradients, without a pump the cell slowly depolarizes until V reaches 0, and $K^{in} = 5$, $Na^{in} = 145$, $Cl^{in} = 110$.

[P.S. — Not shown, but required for electroneutrality, is that $A^- = 146$ inside and $A^- = 40$ outside, where A^- = impermeable anions. Note that this also puts the cell in osmotic equilibrium (300 mosm at start). But, as $K^{in} \rightarrow 5$, $Na^{in} \rightarrow 145$ and $Cl^{in} \rightarrow 110$, internal osmolarity drops as follows:

At final equilibrium, $\frac{K^{in}}{K^{out}} = \frac{Na^{in}}{Na^{out}} = \frac{Cl^{out}}{Cl^{in}}$ (Donnan Rule)

$$\frac{K^{in}}{5} = \frac{Na^{in}}{145} = \frac{110}{Cl^{in}}$$

$K^{in} + Na^{in} = Cl^{in} + A^{in}$ (electroneutrality)

$K^{in} + Na^{in} + Cl^{in} + A^{in} = 300$ (osmotic equilibrium)

$$K^{in} + \frac{145}{5}K^{in} + \frac{110}{5K^{in}} + A^{in} = 300$$

$$K^{in} + \frac{145}{5}K^{in} - \frac{110}{5K^{in}} - A^{in} = 0$$

$$\Rightarrow 2K^{in} + 2 \cdot \frac{145}{5} K^{in} = 300$$

$$\Rightarrow K^{in} = 5,\ Na^{in} = 145,\ Cl^{in} = 110$$

and $A^{in} = 40$

so osm^{in} drops to 40 mM by water entry, cell swells, and lyses!

<u>All this is just an aside...</u>

4.

$$\left.\begin{aligned} I_{Na-p} &= -1.5 I_{K-p} = -r I_{K-p} \\ I_{Na} &= -I_{Na-p} \\ \cancel{I_{Na}} + I_K + I_{Cl} + I_{K-p} + \cancel{I_{Na-p}} &= 0 \end{aligned}\right\} \left. I_K + I_{Cl} + \underbrace{\underbrace{I_{K-p}}_{-\frac{1}{r} I_{Na-p}}}_{+\frac{1}{r} I_{Na}} = 0 \right\} I_{Na} + r I_K + r I_{Cl} = 0$$

Nernst-Planck single Ion Equations:

$$I_i = z_i^2 q \xi P_i \frac{c_i^{in} - c_i^{out} e^{-z_i \xi}}{1 - e^{-z_i \xi}} \quad \text{where } \xi = qV/kT$$

$$I_{Na}\ (z=+1) \quad + \quad + I_K\ (z=+1) \quad + \quad I_{Cl}\ (z=-1) \quad = 0$$

$$\frac{q\xi P_{Na}(Na^{in} - Na^{out} e^{-\xi})}{1-e^{-\xi}} + \frac{rq\xi P_K(K^{in} - K^{out} e^{-\xi})}{1-e^{-\xi}} + \underbrace{\frac{rq\xi P_{Cl}(Cl^{in} - Cl^{out} e^{+\xi})}{1-e^{+\xi}}}_{\underbrace{\frac{(-e^{-\xi})}{(-e^{-\xi})} \cdot \frac{rq\xi P_{Cl}(Cl^{in} - Cl^{out} e^{+\xi})}{(1-e^{+\xi})}}_{\frac{rq\xi P_{Cl}(Cl^{out} - Cl^{in} e^{-\xi})}{1-e^{-\xi}}}} = 0$$

Multiply by $\frac{1-e^{-\xi}}{q\xi}$, collect Terms, divide by P_K

$$\frac{P_{Na}}{P_K} Na^{in} + r\, K^{in} + r\frac{P_{Cl}}{P_K} Cl^{out} = \left(\frac{P_{Na}}{P_K} + r\, K^{out} + r\frac{P_{Cl}}{P_K} Cl^{in}\right) e^{-qV/kT}$$

$$e^{qV/kT} = \left(\frac{P_{Na}}{P_K} Na^{out} + rK^{out} + r\frac{P_{Cl}}{P_K} Cl^{in}\right) \Big/ \left(\frac{P_{Na}}{P_K} Na^{in} + rK^{in} + r\frac{P_{Cl}}{P_K} Cl^{out}\right)$$

$$V = \frac{kT}{q} \ln\left(\frac{\cdots}{\cdots}\right) = 58 \log_{10} \frac{\frac{P_{Na}}{P_K} Na^{out} + rK^{out} + r\frac{P_{Cl}}{P_K} Cl^{in}}{\frac{P_{Na}}{P_K} Na^{in} + rK^{in} + r\frac{P_{Cl}}{P_K} Cl^{out}} =$$

$$= 58 \log_{10} \frac{(0.019)(145) + (1.5)(5) + (1.5)(0.381)(4)}{(0.019)(10) + (1.5)(140) + (1.5)(0.381)(110)} = -77.6\text{ mV} \quad \text{with pump}$$

5. Ouabain: Cell depolarizes by 2.6 mV To −75.0 mV