

## Answer Key - Quantitative Question from Lecture #7

Radionuclides (radioisotopes) are atoms with nuclei that disintegrate by emission of particles (alpha/ $2p^+ - 2n$  or beta/ $1e^-$  rays) or electromagnetic radiation (gamma rays).  $^{32}\text{P}$  is a strong beta emitter (1.71 MeV; for comparison, the beta emission of tritium/ $^3\text{H}$  is only 0.0185 MeV).

Decay of the atoms of a radioisotope is a random first-order process with exponential kinetics; each nucleus has the same probability of decay as any other nucleus, and the decay of any given nucleus does not affect the rate at which any other nucleus will decay. Thus, at any particular time, the number of atoms of a radioactive material decaying per unit time is proportional to the number of atoms present at that time.

Each radionuclide emits at a characteristic rate, different from that of other radionuclides. The rate of radioactive decay of a radioactive nuclide is a constant and, as mentioned, directly proportional to the amount of radioactive material present initially. Therefore, the rate of decay of a radioactive nuclide is most conveniently defined as its "half-life," i.e. the time required for a sample of atoms of a radioisotope to be reduced to exactly one half of the number of radioactive atoms that were present in any starting sample. Half-life is usually symbolized as  $\tau_{1/2}$ . The half-life of a radionuclide is related to its decay constant ( $\lambda$ , in disintegrations per sec), as follows:  $\tau_{1/2} = -(\log_e 1/2) / \lambda = +0.6931 / \lambda$ .

Thus, the number of radioactive atoms remaining ( $N_t$ ) after the passage of time,  $t$ , where the number of radioactive atoms initially was  $N_0$ , is:

$$N_t = N_0 e^{-\lambda t}, \text{ where } e = 2.718$$

(which is sometimes called the "Law of Radioactive Decay")

or, more conveniently, that

$$\text{Percentage of isotope remaining} = N_t / N_0 = 100 \times e^{-0.6931 (t/\tau_{1/2})}$$

You were given that the initial amount of [ $\gamma$ - $^{32}\text{P}$ ]ATP was 10 mCi,\* and that 42 days (4 Dec. to 15 Jan.) passed.  $^{32}\text{P}$  has a half-life of 14.3 days (which you could look up in any number of places).

$$100 \times e^{-0.6931 (42 / 14.3)} = 13.06\%$$

$$10 \text{ mCi} \times 0.1306 = \boxed{1.31 \text{ mCi}}$$

So, by waiting and delaying close to a month-and-a-half, you lost close to 90% of the starting material, and the remaining ATP molecules are probably no longer intact anyway due to the damage (radio-decomposition) they incurred when these molecules were struck by the released high energy beta particles from the decaying molecules, which readily break any chemical bonds they hit.

\*A Curie (abbreviated Ci) is that quantity of a radioisotope required to supply  $3.7 \times 10^{10}$  disintegrations per second, where one dps is now called 1 Becquerel (Bq).