Following the flow of 'stuff'

Follow the money

Most modeling is simply bookkeeping the flows of quantities that can be tracked: mass, number, money, etc. Consider, for example, your savings account. The flow of money (with units $) is diagramed in the **FlowChart** window shown in **Figure 1**.

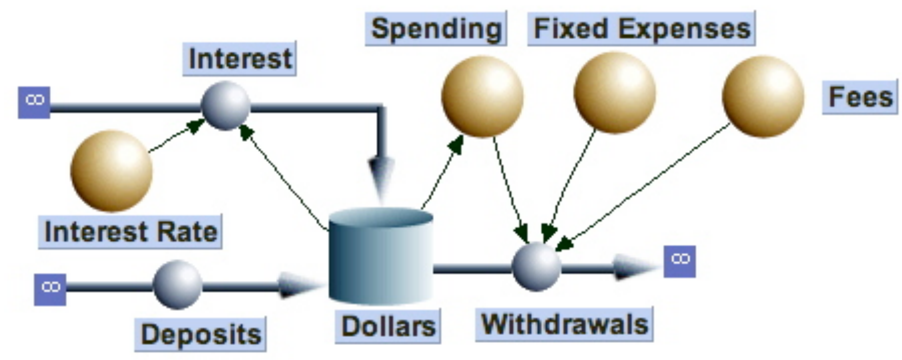

Figure 1. the flowchart following the flow of money in and out of a bank account.

The three icons required to represent the flow of money are:

1. The **Reservoir**  (units = $)

2. The **Flow**  (units = $/t)

3. The **Functions**  show how the flows and reservoir depend on one another. The units of Functions depend on what icon they modify.

4. **Arcs**  connect functions to flows or reservoirs to show dependences.

The **Equation Window** shows the equations corresponding to the Flowchart:

Rate of accumulation:	d/dt (Dollars) = +Deposits - Withdrawals + Interest
Initial Amount: [$}	INIT Dollars = 1000
Inflow: [$/t]	Deposits = 2000
	Interest = Dollars*Interest_Rate
Outflow: [$/t]	Withdrawals = Fixed_Expenses + Fees + Spending

Parameters: [$/t]	Interest_Rate = 0.04 Fixed_Expenses = 900 Fees = 10 Spending = Dollars*0.5

Now, suppose you have a foreign bank account. You can transfer funds from your bank account in England, but you have to convert from pounds (**£**) to dollars (**$**). Suppose you transfer 1% of your British assets every month to your American account. Then the flow of money through the two linked bank accounts is shown in **Figure 2**.

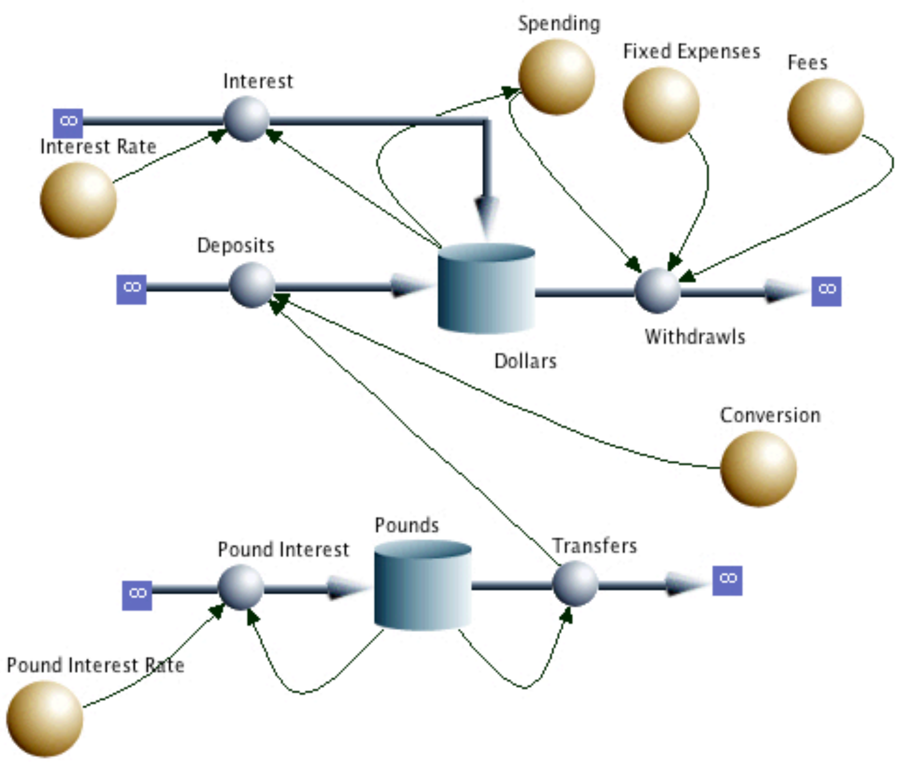

Figure 2. The flow of money in two linked bank accounts

The Equation Window shows the same information in symbolic form:

Reservoirs:	d/dt (Dollars) = + Deposits - Withdrawls + Interest
	INIT Dollars = 1000
	d/dt (Pounds) = + Pound_Interest – Transfers
	INIT Pounds = 100
Flows:	Deposits = Conversion*Transfers+2000
	Withdrawls = Fixed_Expenses+Fees+Spending
	Interest = Dollars*Interest_Rate
	Pound_Interest = Pound_Interest_Rate*Pounds
	Transfers = 0.01*Pounds
Functions:	Fixed_Expenses = 900
	Fees = 10
	Spending = Dollars*0.5
	Interest_Rate = 0.04
	Pound_Interest_Rate = 0.08
	Conversion = 1.5

Flows of physical quantities

The flows of physical quantities are different in a fundamental way from the flow of money in the example above. There the flows were determined by the withdrawal and deposit functions that depend on many external factors, including the whims and psychology of the investor. In most cases, the flow of physical quantities are driven by *potentials* that can be calculated from physical laws. For example, the flow of heat is driven by temperature differences, the flow of fluids are driven by differences in pressure differences, and the diffusion of particles is driven by differences in concentration. As a consequence of the physical origin of driving forces, you will encounter a common Flowchart structure again and again; it has the general form shown in **Figure 3**.

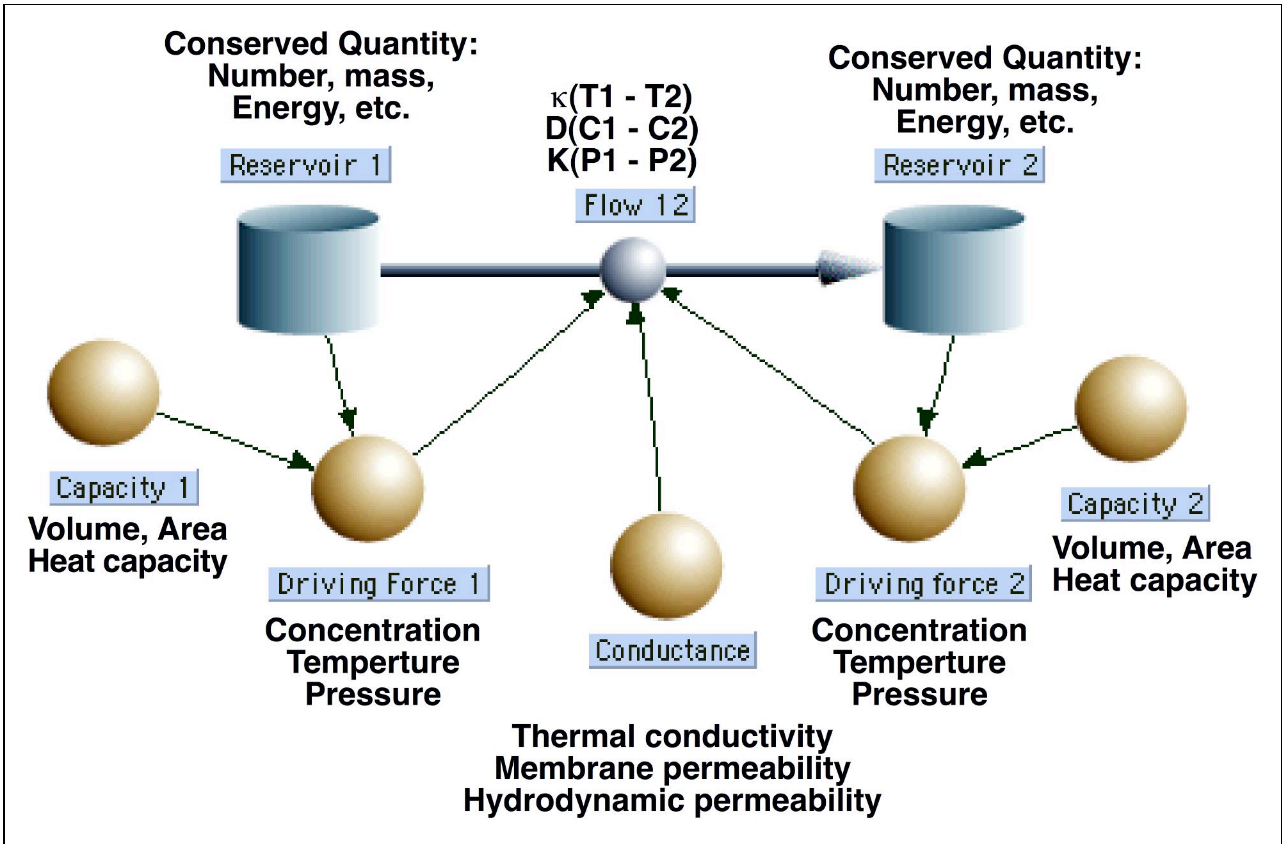

Figure 3. The general diagram for tracking the flow of a conserved quantity. The model equations will have the general form:

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Top model}

  {Reservoirs}
  d/dt (Reservoir_1) =  - Flow_12
     INIT Reservoir_1 = 1
  d/dt (Reservoir_2) = Flow_12
     INIT Reservoir_2 = 1

  {Flows}
  Flow_12 = Conductance*(Driving_Force_1-Driving_force_2)

  {Functions}
  Conductance = 1
  Driving_Force_1 = Reservoir_1/Capacity_1
  Capacity_1 = 1
  Capacity_2 = 1
  Driving_force_2 = Reservoir_2/Capacity_2
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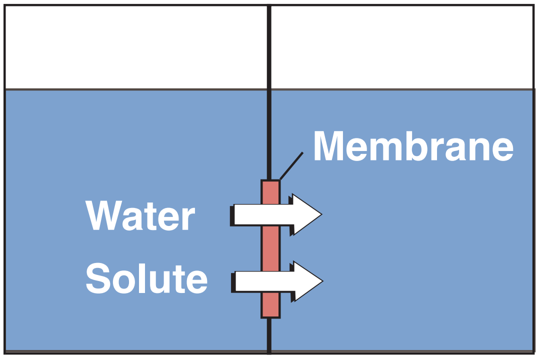
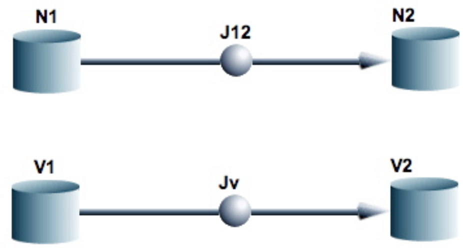
EXAMPLE. Consider the following situation where water flows across a semipermeable membrane in response to an osmotic pressure difference.





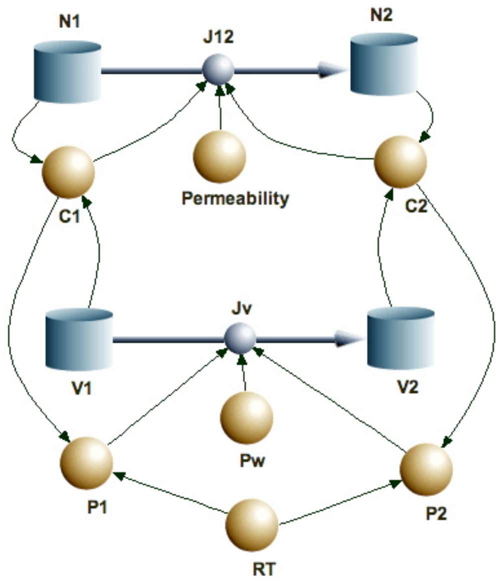
We want to keep track of the flow of solutes and of water. The conserved quantities are **N** = number of solute molecules and **M** = mass of water. However, it is more convenient to scale these quantities:

- # moles of solute: n [mol] = N/N_A, where N_A is Avogadro's number = $6x10^{23}$ molecules/mol.
- Volume of water: V [cm^3] = m [gm]/ρ [gm/cm^3]. Since the density of water ρ ~ 1, the volume is numerically equal to the mass.

The driving force for diffusion of solute across the membrane is Fick's law:

- Diffusive flux of solute = P ($c_1 - c_2$), where c_i = concentration of solute in compartment I = 1, 2, and P is the permeability.
- Molar concentration of solute: c [M] = n/V (M = moles/liter)

The driving force for water flow is the pressure difference: Water flow = $P_V(p_1 - p_2)$, where the pressure difference is computed from van't Hoff's law: $(p_1 - p_2) = RT\cdot(\chi_1 - c_2)$. The Flowchart for this problem is shown below. As the Flowchart is being constructed, the equation window shows the model equations being assembled.



{Top model}
{Reservoirs}
d/dt (N1) = - J12
INIT N1 = 1
d/dt (N2) = J12
INIT N2 = 0
d/dt (V1) = - Jv
INIT V1 = 1
d/dt (V2) = Jv
INIT V2 = 1
{Flows}
J12 = Permeability*(C1-C2)
Jv = Pw*(P1-P2)
{Functions}
Permeability = 1
C1 = N1/V1
C2 = N2/V2
Pw = 1
P1 = RT*C1
P2 = RT*C2
RT = 1